

## STOCHASTIC FRONTIER PRODUCTION FUNCTION: AN APPLICATION TO ANKARA MOHAIR GOAT FARMING SYSTEM

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### Abstract

*The main purpose of this paper is to ascertain changes in the performance of the Mohair (Ankara) goats farming system during the 2017-2018 production periods. In this study, we examined both production and technical efficiency of goat's farms in Ankara. The study adopted the stochastic frontier production function to estimate technical efficiency of the mohair goat establishments in the study area. The model utilized data from 45 Turkish mohair goat farmers. At the same time, technical inefficiency effect model also was calculated by the same method. The Cobb-Douglas production function was ascertained as a best fitted model. In addition, the technical inefficiency effects contained random element. The empirical results revealed statistically insignificant individual effects of some of the variables. However, all of the joint effects of these four explanatory variables on the levels of technical inefficiencies were statistically significant in the mohair goat farming system of Ankara in Turkey.*

**Key words:** stochastic frontier production function, mohair (ankara) goats

### INTRODUCTION

Angora goat was only bred in Anatolia until 1838 when the breeding spread to several other countries, mainly to South Africa. Angora goat is not generally grown in Ankara only, but also in some other provinces such as Konya, Karaman, Kırşehir, Niğde, Yozgat, Eskişehir, Kütahya, Bolu, Afyonkarahisar, Çankırı, Çorum, Kastamonu, Siirt, Mardin, Bitlis and Van. Angora goats are uniquely characterized by a cover of soft body, small, curly and bright mohair. A very important production strategic purpose is for their mohair. The mohair comes in variety of featured colours such as white, cream, yellow, black, gray, brown and black. Some determinant criteria for mohair yield is as follows: gender, age and feeding type of Angora goats. Mohair yield from male goats differs from that of female goats; Yield range from an average of 3 kg in males and 1.5 - 3 kg in females. Mohair is used in different sectors for products such as clothing, blankets, carpets, shawls, hats, scarves, boots, slippers, wigs etc. [9].

*Objectives of this study:* Firstly, the study seeks to explore the technical efficiency effects are searched. Second, if present, whether or not they contain a significant random element. Third, as a matter of importance to question whether technical inefficiency effects are influenced by farm specific factors. Fourth, the other crucial question is to assess the return to scale in mohair farming in Ankara.

This paper contains three sections as follows: section two describes the stochastic frontier model used in this study. Section three presents empirical results and discussion. And, the final section consists of the major conclusions of our research.

### MATERIALS AND METHODS

#### *Stochastic frontiers and efficiency measurement*

Analysis of this data was implemented by the production function frontier approach. The determination of maximum possible output per unit produced given input X [5]. [1], [5] were the first author to emphasize about the stochastic frontier production function.

The subject of production efficiency measurement has been a crucial research area in economics and other scientific subjects. In this study, Stochastic frontier production function has been used. [3] implemented the stochastic frontier production function onto many agricultural studies. [8] emphasized that there are some advantages of stochastic frontier models. These advantages are as follows: Firstly, it presents a disturbance term which stands in for statistical noise, measurement error and exogenous shocks exceeding the control of production units which in other respects, would be attributed to technical inefficiency. Secondly, it renders the underlying foundation for carrying out statistical tests of hypothesis relating to the production structure and the extent of inefficiency. Thirdly, the evaluation of frontier function and efficiency can be accomplished either in one or two stages.

In this paper, [2] approximations were used. One of this approximation is stochastic and the other is effects of technical inefficiency in the frontier function. Estimation of all the parameters is made by the Maximum Likelihood Method that forms a single step estimation analysis in this study. [6] emphasized an approach to efficiency measurement and conducted an empirical analysis from different sampled farmers in Pakistan. [4] estimated the production of the technical efficiencies and implemented their approaches during their study.

**Model and variables**

The study used primary data which were collected from 45 mohair goat farmers/establishments located in the Ankara province [7]. For estimating the parameters, Cobb-Douglas production function, being an adequate model and given the properties of the corresponding frontier model, was used. In this paper, stochastic frontier model is defined as follows:

$$\ln(Y_i) = \beta_0 + \beta_1 \ln(X_{1i}) + \beta_2 \ln(X_{2i}) + \beta_3 \ln(X_{3i}) + \beta_4 \ln(X_{4i}) + \beta_5 \ln(X_{5i}) + V_i + U_i \dots \dots \dots (1)$$

where:

$\ln$ : represents the natural logarithm that is base: e,

The subscript,  $i$  denotes the  $i^{\text{th}}$  farmer in this research sample  $i=1,2,\dots,45$ .

Mohair production, ( $Y_i$ ): the total mohair production for the farmer,

Employee wage ( $X_{1i}$ ): the total employee;

Forage ( $X_{2i}$ ); the quantity of forage;

Veterinary ( $X_{3i}$ ): the total cost of veterinary applied to all of he goat;

Vaccine-medicine ( $X_{4i}$ ): the total cost of vaccine-medicine

Other costs ( $X_{5i}$ ): the other remains total costs.

The  $\beta_k$ 's:  $k=0, 1, 2, 3, 4, 5$  are undisclosed parameters for the production function.

The  $V_i$ 's: represents random errors associated with errors of measurement in the production of mohair as reported, or the combined effects of input variables which are not included in the production function, where  $V_i$ 's are assumed to be independently and identically distributed  $N(0, \sigma_v^2)$  random variables.

The  $U_i$ 's represent non-negative random variables that are associated with technical inefficiency of production by the farmers which are assumed to be independently distributed in such a way that the technical inefficiency effect for the  $i$ -th farmer.

$U_i$  is obtained by truncation at zero of the normal distribution with mean  $U_i$  and variance  $\sigma^2$  such that,

$$U_i = \delta_0 + \delta_1 Z_{1i} + \delta_2 Z_{2i} + \delta_3 Z_{3i} + \delta_4 Z_{4i} \dots \dots \dots (2)$$

where:

$Z_{1i}$  : the state support,

$Z_{2i}$  : marketing,

$Z_{3i}$  : stud animals,

$Z_{4i}$  : animals disease,

$\delta_s$  : unknown parameters to be estimated.

Frontier 4.1 software program was written by [3].

It was used to estimate for the stochastic frontier model. Estimated variance parameters from the frontier model are  $\sigma_s^2 = \sigma_v^2 + \sigma^2$  and  $\gamma = \frac{\sigma^2}{\sigma_s^2}$  and here the  $\gamma$  parameter has values ranging between zero and one.

## RESULTS AND DISCUSSIONS

The parameters of the stochastic frontier production function are estimated using the

maximum likelihood estimation. According to the equation 1 and 2 are calculated and these results are given in here standard errors and t-values in Table 1.

Table 1. Parameters of Stochastic Frontier Production Function and Inefficiency model for Mohair Goat Farmers in Ankara, Turkey

| Variable                | Coefficient | Standard-error | t-value |
|-------------------------|-------------|----------------|---------|
| $\beta_0$               | 0.298       | 0.967          | 0.309   |
| Ln of Employee wage     | -0.222      | 0.633          | -0.351  |
| Ln of Forage            | 0.106       | 0.111          | 0.955   |
| Ln of Veterinary        | -0.418      | 0.896          | -0.467  |
| Ln of Vaccine-medicine  | 0.567       | 0.948          | 0.598   |
| Ln of Other costs       | 0.613       | 0.199          | 0.309   |
| Inefficiency model      |             |                |         |
| $\delta_0$              | -0.245      | 0.161          | -0.152  |
| The state support       | 0.325       | 0.107          | 0.306   |
| Marketing               | -0.220      | 0.159          | -0.138  |
| Stud animals            | -0.273      | 0.944          | -0.289  |
| Animals disease         | -0.111      | 0.358          | -0.311  |
| Variance parameters     |             |                |         |
| $\sigma_s^2$            | 0.733       | 0.192          | 0.382   |
| $\gamma$                | 0.999       | 0.169          | 0.593   |
| Log-likelihood Function | -54.79      |                |         |

Source: Authors' results.

### Stochastic Frontier:

$$\ln Y = 0.298 - 0.222 \ln(\text{Employee wage}) + 0.106 \ln(\text{Forage}) - 0.418 \ln(\text{Veterinary}) + 0.567 \ln(\text{Vaccine medicine}) + 0.613 \ln(\text{Other costs})$$

$$U_t = -0.245 + 0.325 \text{The state support} - 0.220 \text{Marketing} - 0.273 \text{Stud animals} - 0.111 \text{Animals disease}$$

### Variance Parameters:

$$\sigma_s^2 = 0.733, \gamma = 0.999$$

### Inefficiency model:

$$\text{Log(likelihood)} = -54.79$$

Table 2. Tests of hypothesis for parameters of the inefficiency frontier model for Mohair Goat Farmers in Ankara, Turkey

| Null Hypothesis  | Log (Likelihood) | $\chi^2_{0.95-value}$ | Test Statistic* | Decision       |
|--|------------------|-----------------------|-----------------|----------------|
| $H_0: \gamma = \delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$ | -38.79           | 11.07                 | 66.13*          | $H_0$ Rejected |
| $H_0: \gamma = 0$  | -49.86           | 9.49                  | 39.91*          | $H_0$ Rejected |
| $H_0: \delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$          | -36.11           | 9.49                  | 71.49*          | $H_0$ Rejected |

Source: Authors' results.

The best fit function for data was found to be The Cobb-Douglas production function taking into account the specification of the corresponding Translog production function model. Log-Likelihood function value for The Cobb-Douglas model was 54.79. Due to the fact that Cobb-Douglas production function is

used in model,  $\beta$  estimates shows the elasticities of production. The estimate of the variance  $\gamma = \frac{\sigma^2}{\sigma_s^2}$  linked with the inefficacy effect is nearly 99.9% of the two variances that it has high value.

The first null hypothesis is tested under the equation as follows:

$$H_0: \gamma = \delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$$

This equation shows the omit of  $u_i$  from the exact model that it implies the restrictions on the full model. When the implementation of restriction on the full model (As shown in equation 1 and 2), log likelihood statistics reduces -38.79. Test statistics value has 66.13 and it is bigger than the critical value 11.07. We can give a decision as a reject for the first restriction. The first null hypothesis is rejected.

The results of hypothesis test of the stochastic frontier parameters and inefficiency effects models for mohair goat farmers in Ankara district are given in Table 2. In this Table 2, the first column belongs to the restriction imposed, that is the null hypothesis. And, the second column shows log likelihood statistics and we can define this column as imposed on the original model. Test statistics are shown in the third column. Critical values for the test statistic are presented in the fourth column. The last column represents the decision column indicating that null hypothesis tested is rejected or not. That is; whether restrictions are valid or not.

The second null hypothesis is given in Table 2 as follows:  $H_0: \gamma = 0$  which shows that technical inefficiency effects aren't stochastic. Given  $\gamma$  is zero, the variance of the technical inefficiency effect is zero. We can conclude from this extraction that the model reduces the traditional mean response function where the state support, marketing, stud animals and animals disease variables are included in the production function. Moreover, the second null hypothesis is equals to zero and then  $\delta_0$  parameter is not identified. Namely, we can say that, the production function has an intercept value. If we implement the restriction on the model ( $H_0: \gamma = 0$ ), log likelihood statistics is becomes -49.86. A generalized likelihood ratio test statistic of 39.91, this value is larger than the critical value of 9.49. In conclusion, the second null

hypothesis is rejected and the technical inefficiency effects aren't random.

The four agricultural specific factors which were included in the inefficiency model have a crucial influence on the degree of technical inefficiency linked with the mohair goat farmers. Null hypothesis is given below:

$$H_0: \delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$$

When this restriction was imposed on the full model, the log likelihood statistics reduces to -36.11. Test statistics belonging to likelihood ratio is was 71.49. This value is greater than the critical value of 9.49. That is to say, the four agricultural specific factors do not have any effects on the technical inefficiency. For this reason, we can reject the null hypothesis.

## CONCLUSIONS

Different restrictions were implemented on the model by equation 1 and 2. Likelihood ratio tests were very crucial in deciding to control whether the restrictions are valid or invalid.

While the individual effects of some of the variables aren't statistically significant, the joint effect of these four explanatory variables on the levels of technical inefficiencies were statistically significant.

In a stochastic frontier production function, technical inefficiency effects were estimated. An application of this model was implemented by using data from 45 Turkish mohair goat farmers in Ankara Province of Turkey. The model for the technical inefficiency effects indicated a significant signs in the stochastic frontier production function. After estimating this model, we can say that inefficiency effects are both stochastic and have a known distribution.

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