

# COMPLEX NETWORK PROPOSAL FOR THE ANALYSIS OF TERRITORIAL EDUCATIONAL COVERAGE IN RURAL AREAS OF SPAIN

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## Abstract

*In this article we will use the approach and methodology of complex networks to analyze the territorial distribution of the public offer of secondary education in Asturias, based on the different population centers, which are the different nodes of the network, and the educational services that can connect them. Through the analysis of its structure and network topology, we will extract information that can be part of and incorporated into a complex socio-technological system that helps us understand the dynamics of Asturian territorial development. We will pay a special attention to the urban or rural nature of the nodes of the network and the implications in this regard that we can observe, due to the condition that the availability of the educational offer supposes for the development of the towns and their inhabitants. The results show a network with a scale-free topology that gives rise to consider possible alternatives that are more balanced from a territorial point of view.*

**Key words:** complex network, public offer of secondary education, network structure measures, topological analysis

## INTRODUCTION

We will define a complex network structure based on the territorial distribution of population units for a specific territory (Principality of Asturias, in Spain), in which the underlying network-shaped structure allows us to describe the interactions between the elementary units that we are going to consider as its support. This will help us to try to determine in turn which are the emergent properties, inherent to any complex system, that we can use, to verify, from a sustainability approach, whether certain land management guidelines are efficient and necessary. We will focus on public educational resources, of a universal and almost free nature, that the corresponding administrations, endowed with the necessary skills and financial resources, use to cover the educational needs of a specific territory.

We will start with a brief review of the literature in which the complex approach is used both to study the relationships or connections between towns and cities, and to study different aspects of the educational

system. Next, the methodology used and our proposal for a complex network are described, to later present the results obtained by analyzing its structure and topology. Finally, we will present the conclusions and a brief discussion about the implications and potential of future lines of research that are based on the proposed approach and the results obtained.

### Brief literature review

There are several authors who have used the complex network approach to analyze the territorial structure of aspects or variables that make it possible to establish connections between different towns or cities distributed spatially in specific territories. In logistics or transportation areas, we can highlight [11] that detect characteristics of a small world network and not free scale in the Chinese air transport network, with a degree distribution that better fits an exponential function, or [2] who have proposed a complex approach to collect the transport system of the islands of Sicily and Sardinia, which has also shown characteristics of a small world network. About telecommunications, [10] have studied the differences between the telecommunications

networks of the United States and Europe, analyzing their topology to see if they conform or not to scale free networks, being distributed or not with a power law. The differences found, the authors understand that should be understood by technological, economic, cultural and political factors that explain the dynamics of these networks. [7] have studied the daily migratory flows of urban China, to determine the existing urban regions based on the connectivity of cities, beyond physical or administrative borders.

On the other hand, the analysis of the validity of the complexity approach to study different aspects of the educational system has also been addressed by different authors.

[6] consider it essential to take advantage of complexity science to understand the properties of educational systems in changing contexts. Also, [4], who concludes that it is necessary to advance in the development of methodological tools in the field of complexity for their applicability to the study of educational systems. Along the same lines, [5] determine the importance of the complex approach, both at a conceptual and methodological level, to advance in the study and understanding of both the educational system and the different educational policies implemented or to be implemented. [8] also shows the adequacy of complexity theory to the field of social sciences and specifically to the field of education.

Our proposal uses an approach based on a combination of the lines of research described, defining a complex network whose nodes are spatially distributed towns and cities and considering some aspects of the educational system as a complex system to try to understand its properties, characteristics, and functioning.

## MATERIALS AND METHODS

### Construction of the network structure

The nodes are determined by the population units, that is, the population centers defined by the gazetteer in the official statistics databases (National Institute of Statistics). Towns and cities configure population accumulations with

a clearly defined historical and socioeconomic structure.

Number of population units (N)=6,942.

We will consider that there is a link when there is a situation between two populations in which one of them offers an educational service of secondary education and any potential student residing in another and meeting the necessary administrative requirements could choose to enroll and study at the corresponding center if desired, as long as the distance allows.

Once we have proposed the network, we will analyze its structural properties and topology, for its analysis, including a statistical contrast to verify if there is similarity with a topology adjusted to a power law, typical of scale free networks, which allows us to approximate its operation.

The source of statistical information from which the data have been obtained is the National Institute of Statistics of Spain (INE) [9]. For the construction and analysis of the proposed complex network, R has been used.

## RESULTS AND DISCUSSIONS

### Structural properties of the network

To analyze the structure of the network we will study, together with the number of edges and nodes, the degree of the nodes and the degree distribution, the average degree, the density of edges, the average length of the paths, the diameter and the clustering coefficient.

The number of vertices is 6,942 and the number of edges is 43,630. The network density in a complex network is the number of edges made between possible edges. In our network it is very low, specifically 0.001810959, since only 39 nodes provide the educational service to the rest. It is the clear result of the centralization in the largest population centers of educational services towards the rest, which allows us to consider education as a public service provided to rural areas from the most urban ones. There are also exceptions, as in the case of Luces, located in a small village, but it is an agrarian training center that needs to have appropriate facilities and farms. In addition, it has a boarding school, so it is an educational service offered to any other nucleus, no matter how distant it is from

the center. In this case, the rural-urban relationship would be the opposite, and would derive from an alternative model to the usual one.

Given  $G = (V, E)$ , we will define the degree of a node,  $v_i$ , as the number of links or edges it has with other nodes

$$\text{deg } v_i = |\{e_{ij} \in E: j \neq i\}|$$

where  $|\cdot|$  denotes cardinality.

We will also define the degree distribution as a percentage, specifically the percentage of network nodes with a given degree,  $k$ . This definition allows it to be interpreted as the probability that a randomly selected network node has  $k$  links.

$$P(k) = \frac{|\{v_i \in V: \text{deg } v_i = k\}|}{|V|} \dots \dots \dots (1)$$

where  $k \in \{0, 1, \dots, \infty\}$ .

The average degree of the network, which we denote by  $\bar{k}$ , is defined as the average of the degrees of all nodes in the network.

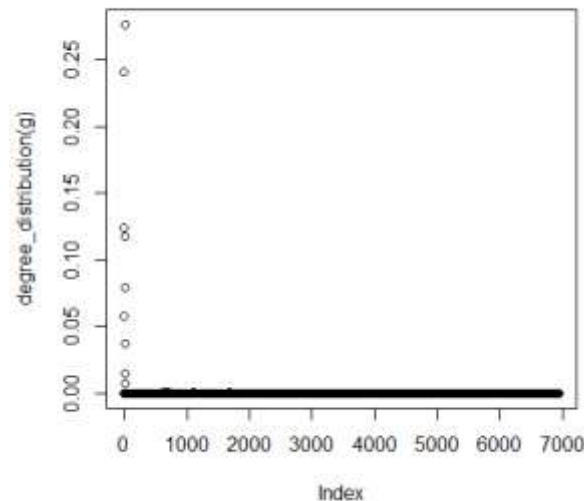


Fig. 1. Degree distribution  
 Source: Own elaboration.

The degree distribution is represented in Figure 1 and is typical of a network with a few highly connected nodes and the rest with few connections, like those known as scale free. These networks also present high heterogeneity and follow a power law in the

distribution of degrees [1], compared to other networks that follow exponential laws and are more homogeneous.

The mean degree of the nodes is 12.56986. We define the local clustering coefficient. According to [13] it is the number of links that a node has among the maximum number of possible links of that node. If the network is undirected, its expression is:

$$C_i = \frac{E_i}{\frac{k_i(k_i-1)}{2}} \dots \dots \dots (2)$$

If we consider it in global terms, that is, for the entire network, the average of the individual clustering coefficients of all the nodes in the network would be the global clustering coefficient.

$$C = \frac{1}{N} \sum_{i=1}^N C_i \dots \dots \dots (3)$$

An equivalent alternative for the calculation of the coefficient of clustering or transitivity would be the one that uses the concept of triplets.

Global clustering coefficient [12], also known as transitivity of the network, indicates the tendency to join between nodes forming groups. It is based on the concept of triplets, which are three nodes connected by two or three edges. If there are two bonds the triplet is open and if there are three the triplet is closed and is called a triangle.

$$C = \frac{N^{\circ} \text{ of closed triplets}}{N^{\circ} \text{ of closed triplets} + N^{\circ} \text{ of open triplets}} \dots \dots \dots (4)$$

For local the expression based on triplets would be

$$C_i = \frac{N^{\circ} \text{ of closed triplets containing node } i}{N^{\circ} \text{ of triplets centered on node } i} \dots \dots \dots (5)$$

In our case, the global coefficient is very low, specifically 0.007193096, because the tendency of nodes to group together is low, since only a few acts as head and are those that provide educational services to the rest.

In the local coefficients, the transitivity of the nodes that do not provide educational service is very high, but very low in those that do, that is, in those that have a secondary school. In the latter, the degree of transitivity is less than 0.03, while in the rest it is mostly 1, and in any case always greater than 0.46. Something that happens because the nodes that have a secondary school have many neighbors and therefore a high degree, unlike the rest.

Average Path Length:

$$A_{length} = \frac{1}{N(N-1)} \sum_{i,j} d(v_i, v_j) \dots \dots \dots (6)$$

For any two nodes we can determine the shortest path connecting them. The number of links in said shortest path is the distance  $d(v_i, v_j)$  between two nodes ( $v_i$  and  $v_j$ ). In our network, the average path length is 1.998204, practically equal to the diameter (maximum of the shortest roads) of the network, which is 2.

$$diam(G) = \max \{d(v_i, v_j)\} \forall v_i, v_j \in V$$

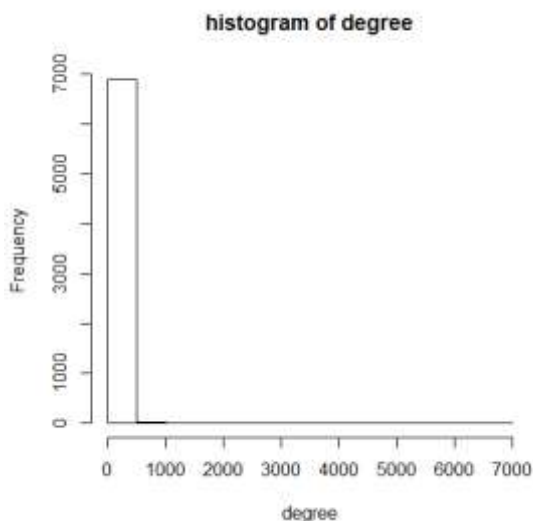


Fig 2. Histogram of degree  
 Source: Own elaboration.

Table 1. Topology of network

<b>Number of nodes</b>	<b>6,942</b>
Number of edges	43,630
Density	0.001810959
Mean Degree	12.56986
Global clustering coefficient	0.007193096
Average path length	1.998204
Diameter	2

Source: Own elaboration.

### Analysis of the network topology

#### Statistical comparison between the observed network and random networks

In principle, our network has a few nodes with many links and most nodes with very few links, so we expect similarities with a scale-free network. These networks have a topology with a degree distribution that conforms to a power law, instead of an exponential law, more typical of Small World type networks, or a Poisson's law, typical of Erdős-Renyi networks [3].

Power Law Topology:  $P(k) = Ck^{-\gamma} \dots \dots \dots (7)$

Exponential Topology:  $P(k) = Ce^{-\alpha k} \dots \dots \dots (8)$

Poisson Topology:  $P(k) = e^{-z} \frac{z^k}{k!} \dots \dots \dots (9)$

For our analysis, we will carry out a fit of the data set to simulate a scale free network and we will compare the original data with said adjustment, to verify, by means of a statistical contrast, whether they follow a power law. The fit is carried out by the method of maximum likelihood.

The Kolmogorov-Smirnov test comparing the fitted distribution with the input vector has a test statistic of 0.005755396. Low values of the same denote better fit, as in this case.

The null hypothesis raised in the test considers that the original data could have been extracted from the distribution adjusted to the power law. In our case, a p-value of 0.9753968 is obtained, when it would need to be less than 0.05 in order to reject the null hypothesis.

The results obtained seem to indicate that the structure and topology of our complex network, which includes the provision of public educational services in the territory of action, contains a few “hubs”, that is, nodes with high connections and a behavior typical of a scale free network. This may raise the possibility of considering the convenience or not, of other alternative distributions, in which the public educational offer of secondary schools contemplates a more equitable development in the territory and with a lower incidence in the more urban centers in favor of rural areas, acting in a complementary way to existing centers, taking advantage of the

possible advantages of online training, not face-to-face and the creation of more centers with internships in cases where the subjects require compulsory attendance.

## CONCLUSIONS

The spatial distribution of basic services, given the possibilities offered by new technologies and educational innovation, could be the object of evolution to a more balanced territorial planning system that requires multidisciplinary study and research for its development. The network distribution, with more distributed connections, without the concentration in a few nodes, would mean in our proposal a territorial rebalancing of a basic and fundamental service such as education, especially when it comes to fixing population in rural areas, thus avoiding the flight of young talent, and encouraging greater involvement of students in their hometowns. There is currently no relevant literature on this initiative, which we believe should be approached from fields related to public economics, pedagogy, operational research, geography or sociology, among others, to lead to a specific proposal for rural and local development, but applicable to any territory that has been granted educational policy competencies, which makes this proposal an initiative of general interest.

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